

Nonabelian Fields in Exact String Solutions

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Abstract

Within the framework of "anomalously gauged" Wess-Zumino-Witten (WZW) models, we construct solutions which include nonabelian fields. Both compact and noncompact groups are discussed. In the case of compact groups, as an example of background containing nonabelian fields, we discuss conformal theory on the $SO(4)/SO(3)$ coset, which is the natural generalization of the 2D monopole theory corresponding to the $SO(3)/SO(2)$ coset. In noncompact case, we consider examples with $SO(2,1)/SO(1,1)$ and $SO(3,2)/SO(3,1)$ cosets.

1 Introduction

The interest in exact string solutions for gravitational backgrounds, most of which were obtained by using conformal field theories, has increased in the last years (for a review see [1] and references therein). The principal motivation for these studies was the hope that these solutions could clarify some aspects of black hole physics and would shed light on relation between the world-sheet and the target-space dynamics. Gagged WZW models play a significant role in construction of exact solutions. Recently a class of theories based on gauge extensions of the WZW models has been introduced [2] in which the gauge anomaly depends only on the gauge field and is independent of a group element; furthermore, the anomaly has the form of the two-dimensional chiral fermion anomaly [3]. The anomaly resulting from the

gauged WZW model is then cancelled against the one-loop anomaly of the fermions. The right sector of the model can be made supersymmetric by an appropriate choice of gauge couplings, left fermions provide current algebra. In [4] it was shown that the monopole construction of paper [5] can be interpreted in terms of the anomalously gauged WZW model.

In this paper the heterotic coset model technique is used to generate backgrounds including nonabelian fields. In sect. 2 we review the heterotic coset model approach and introduce a suitable parametrization for the $SO(3)/SO(2)$ coset. In sect. 3 we present a similar construction for the $SO(4)/SO(3)$ coset. The gauge group $SO(3)$ being nonabelian, the resulting backgrounds are shown to include nonabelian gauge fields. In sect. 4 using parametrization of a group element similar to that used in compact case, we consider examples of conformal theories on $SO(2,1)/SO(1,1)$ and $SO(3,2)/SO(3,1)$ and cosets. In sect. 5 we make some concluding remarks.

2 Heterotic Coset Model Technique on the Example of $SO(3)/SO(2)$ Coset

In this section we briefly describe construction of heterotic coset models and discuss potential problems that appear in the case of nonabelian subgroups which are needed to introduce nonabelian fields as a part of backgrounds. As is standard in the heterotic coset model construction, we begin with an anomalously gauged WZW model. In this paper we discuss the right gauged WZW models, but further generalization is evident. The right gauged model is given by the following action:

$$\begin{aligned}
I_{RGWZW}(g, A) &= kI(g) + \frac{k}{2\pi} \int_{\Sigma} d^2z \text{Tr} A_{\bar{z}} g^{-1} \partial_z g - \frac{k}{4\pi} \int_{\Sigma} d^2z \text{Tr} A_z A_{\bar{z}} \\
I(g) &= \frac{1}{4\pi} \int_{\Sigma} d^2z \text{Tr} \partial_{\mu} g \partial^{\mu} g^{-1} \\
&+ \frac{1}{6\pi} \epsilon^{ijk} \text{Tr} \int_{B, \partial B = \Sigma} d^3y g^{-1} \partial_i g g^{-1} \partial_j g g^{-1} \partial_k g,
\end{aligned} \tag{1}$$

here $g \in G$, $A_a \in H \subset G$, k is the level of the Kac-Moody algebra. Under the gauge transformations

$$g \rightarrow gh \quad \delta g = -gu \quad \delta A_a = -D_a u = -\partial_a u - [A_a, u], \quad (2)$$

the action changes as

$$\delta I(g, A) = \frac{k}{4\pi} \int d^2 z \text{Tr} u (\partial_z A_{\bar{z}} - \partial_{\bar{z}} A_z) \quad (3)$$

Since this expression has exactly the same structure as the nonabelian chiral anomaly, we can add to the action $I(g, A)$ a set of fermions and impose the condition of anomaly cancellation. Still we have to keep in mind that since the anomaly (3) of the gauged WZW model is classical, and the anomaly from the fermionic sector appears at the one-loop level, the anomaly cancellation condition relates two different coupling constants. To represent a resulting theory in a form of a gauge invariant classical theory, we have to bosonize the fermionic sector. As an example, let us consider conformal theory on the $SO(3)/SO(2)$ coset. As a starting point for further generalizations, we use parametrization of the group element as in [6]¹. In the general case of $SO(d)/SO(d-1)$ cosets, the group element of $SO(d)$ can be represented as $g = th$ ², where $h \in SO(d-1)$ and $t \in SO(d)/SO(d-1)$. For the case of interest, the right gauged WZW, a natural gauge choice is $h = 1$ for which $g = t$. The matrix t is of the form:

$$t = \begin{pmatrix} b & (b+1)\hat{X}^\nu \\ -(b+1)\hat{X}_\mu & \delta_\mu^\nu - (b+1)\hat{X}_\mu \hat{X}^\nu \end{pmatrix},$$

here \hat{X}^μ is a row (x^1, x^2) , \hat{X}_μ is a column composed of the same elements, $b = (1 - x^2)/(1 + x^2)$. The indices are contracted with the Euclidean metric $\delta_{\mu\nu} = \text{diag}(1, 1)$. Introducing two right-moving fermions which play the role of the superpartners of the fields x^1 and x^2 and a pair of left-moving fermions coupled to the gauge field with the strength Q , one obtains the anomaly cancellation condition, which has the same form as in the monopole theory [4],

$$k = 2(Q^2 - 1) = 2Q_+ Q_-,$$

¹In contrast to the commonly used Euler parametrization of the $SO(3)$ group, this parametrization can be generalized to the case of higher dimensional groups.

²To make the relation with the form of element g of [6] explicit, we have to identify g with g^{-1} .

here $Q_{\pm} = Q \pm 1$. The coupling for the right gauge field is fixed by requiring supersymmetry in the right sector. The gauge field in equation (1) is of the form

$$A_z = a_z T = \frac{1}{2} a_z \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix},$$

where T is the generator of the gauged subgroup $SO(2) \subset SO(3)$, and the same relation holds for $A_{\bar{z}}$ and $a_{\bar{z}}$. The next step of this construction is bosonization of the fermionic sector to obtain a classical gauge invariant action. The bosonic action is required to yield the same classical "anomaly" as the (non)abelian one-loop fermionic anomaly and is taken in a form [4]

$$I_B = \frac{1}{4\pi} \int_{\Sigma} d^2 z [(\partial_z \Phi - Q_+ a_z)(\partial_{\bar{z}} \Phi - Q_+ a_{\bar{z}}) - Q_- \Phi F_{z\bar{z}}]. \quad (4)$$

Under the transformation

$$\delta A_a = \partial_a u \quad \delta \Phi = Q_+ u,$$

the bosonic action yields classical anomaly of the form

$$\delta I_B = \frac{1}{4\pi} \int_{\Sigma} d^2 z Q_+ Q_- F_{z\bar{z}}.$$

which reproduces fermionic anomaly³.

The same action (4) was used in [4] for the treatment of the monopole theory on the $SU(2)/U(1)$ coset. At this point a remark is needed. Supersymmetry in the right sector implies that the gauge field which gauges the right symmetry in the WZW action must be used also for gauging the right fermions which are the superpartners of the group-manifold coordinates. For our case, we need two right fermions, the number of left fermions is not constrained, but if we keep in mind that we are going to bosonize the fermions using a WZW model, we must take the same number of right and left fermions, because one bosonic degree of freedom is represented by two fermionic fields. Strictly speaking, the action (4) is not the bosonized fermionic action, because the superpartners must belong to the coset $SO(3)/SO(2)$, and we must

³Another bosonic action which yields the same anomaly is obtained by interchanging Q_+ and Q_- in (4). It can be verified that the final form of backgrounds is independent of this ambiguity.

use nonabelian bosonization or, in other words, use a gauged WZW model defined on $SO(3)$. What makes the above choice (4) admissible, is the assumption [4] that the final metric must depend only on coordinates of the bosonic part of the original theory which we are going to interpret as the coordinates of the physical space. The bosonic action (4) is no longer suitable for nonabelian cosets, in which case the bosonized fermions must be represented as an anomalously gauged WZW at level 1. In the next section we will show that this fact results in considerable complications in the problem of reading off the metric.

Collecting (1) and (4), we get the total action

$$\begin{aligned}
I^{total} &= kI(g) + \frac{k}{8\pi} \int_{\Sigma} d^2z \frac{2}{k} \partial_z \Phi \partial_{\bar{z}} \Phi \\
&+ \frac{k}{8\pi} \int_{\Sigma} d^2z \left(a_{\bar{z}} (4J_z - 2 \frac{(Q_+ - Q_-)}{k}) \partial_z \Phi \right. \\
&- \left. a_z \frac{2(Q_+ + Q_-)}{k} \partial_{\bar{z}} \Phi + a_z a_{\bar{z}} (2 \frac{Q_+^2}{k} + 1) \right). \tag{5}
\end{aligned}$$

Introducing new variables

$$x^1 = r \cos \phi \quad x^2 = r \sin \phi, \tag{6}$$

the term $I(g)$ and the current J_z can be written as

$$\begin{aligned}
I(g) &= \frac{2}{\pi} \int_{\Sigma} d^2z \left(\frac{1}{(1+r^2)^2} \partial_z r \partial_{\bar{z}} r + \frac{r^2}{1+r^2} \partial_z \phi \partial_{\bar{z}} \phi \right) \\
J_z &= -\frac{2r^2}{1+r^2} \partial_z \phi. \tag{7}
\end{aligned}$$

Integrating out the gauge fields, we obtain the dilaton shift at the one-loop level, which, in the present case, is trivial because it is independent of any field variable and is equal to $\ln(2Q_+^2 + k)$. For nonabelian cosets and even for other gaugings, the behavior of the dilaton is completely different. The resulting action is:

$$I^{total} = kI(g) + \frac{k}{8\pi} \int_{\Sigma} d^2z \left(\frac{1}{Q_+^2} \partial_z \Phi \partial_{\bar{z}} \Phi + \frac{4}{Q_+} J_z \partial_{\bar{z}} \Phi \right) \tag{8}$$

To read off the metric, we still have to refermionize this action. The naive procedure for reading off the metric of sigma models cannot be applied here

because the constants k and Q are connected by the anomaly cancellation condition, and thus the fermionic sector affects the metric. The method proposed in [4] to extract the low-energy background consists in rewriting the action (8) in a symmetric form which prepares refermionization of the action. This uniquely determines the change of the metric of what we interpret as the bosonic sector of the heterotic sigma model. Following this recipe, we have

$$\begin{aligned}
I^{total} &= kI(g) \\
&+ \frac{k}{8\pi} \int_{\Sigma} d^2z \left(\frac{1}{Q_+^2} (\partial_z \Phi + 2Q_+ J_z) (\partial_{\bar{z}} \Phi + 2Q_+ I_{\bar{z}}) \right. \\
&\quad \left. - 4J_z I_{\bar{z}} + \frac{2}{Q_+} (J_z \partial_{\bar{z}} \Phi - \partial_z \Phi I_{\bar{z}}) \right), \tag{9}
\end{aligned}$$

where $I_{\bar{z}} = Tr T g^{-1} \partial_{\bar{z}} g$. This form of the action makes the way the metric is affected by fermions explicit. Now it is possible to associate to the total action the following heterotic sigma model action

$$\begin{aligned}
I^{total} &= \frac{k}{2\pi} \int_{\Sigma} d^2z \left(G_{ij} \partial_{\mu} x^i \partial^{\mu} x^j \right. \\
&\quad + Tr \lambda_R (\partial_{\bar{z}} + A_i \partial_{\bar{z}} x^i) \lambda_R \\
&\quad + Tr \lambda_L (\partial_z + Q A_i \partial_z x^i) \lambda_L \\
&\quad \left. + F \lambda_R \lambda_R \lambda_L \lambda_L \right), \tag{10}
\end{aligned}$$

where the backgrounds are:

$$\begin{aligned}
G_{rr} &= \frac{1}{(1+r^2)^2}, \\
G_{\phi\phi} &= \frac{r^2}{1+r^2} - \frac{r^4}{(1+r^2)^2}, \\
G_{\phi r} &= 0, \\
A_{\phi} &= 2 \frac{r^2}{1+r^2}, \quad A_r = 0, \\
F &= \frac{1}{Q_+}. \tag{11}
\end{aligned}$$

To make contact with calculations of Johnson [4], we must make the following change of coordinates:

$$r = \tan \theta,$$

In these coordinates the part of the metric from $I(g)$ is

$$ds_0 = d\theta^2 + \sin^2 \theta d\phi^2.$$

The modification of the metric resulting from the fermions is

$$-\frac{r^4}{(1+r^2)^2}d\phi^2 = -\sin^4 \theta d\phi^2$$

Finally, rescaling $\theta \rightarrow \frac{\theta}{2}$, we obtain the background in the same form as in ref. [4]:

$$\begin{aligned} ds^2 &= d\theta^2 + \sin^2 \theta d\phi^2 \\ A_\phi &= 2 \sin^2 \frac{\theta}{2}, \quad A_\theta = 0. \end{aligned} \tag{12}$$

In these coordinates, it is clear that the metric describes the two-dimensional monopole theory. As we shall see in the next section, an advantage of the parametrization we have used here is that the matrix form of the group element is easily generalized for higher dimensions. For higher-dimensional groups the form of the group element expressed in Euler angles is very complicated, even for the group $SO(4)$ construction of the group element in Euler parametrization is very involved.

3 Nonabelian Fields from Coset $SO(4)/SO(3)$

In this section we apply the technique described in the previous section to obtain backgrounds with nonabelian fields.

As before, the group element is parametrized as $g = th$, where $X^\mu = (x^1, x^2, x^3)$. As in section 2, we make transformation to spherical coordinates according to

$$\begin{aligned} x^1 &= r \cos \theta \\ x^2 &= r \sin \theta \cos \phi \\ x^3 &= r \sin \theta \sin \phi. \end{aligned} \tag{13}$$

In these coordinates we have (a general property of this type of cosets is that the Wess-Zumino term is zero):

$$I(g) = \frac{2}{\pi} \int_{\Sigma} d^2 z \frac{1}{1+r^2} \left(\frac{1}{1+r^2} \partial_\mu r \partial^\mu r \right)$$

$$+ r^2(\partial_\mu \theta \partial^\mu \theta + \sin^2 \theta \partial_\mu \phi \partial^\mu \phi). \quad (14)$$

The left-moving currents are defined as $J_z^a = \text{Tr} T^a g^{-1} \partial_z g$, where T^a are the generators of the gauge subgroup $SO(3)$ used in this representation:

$$\begin{aligned} T^1 &= \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\ T^2 &= \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \\ T^3 &= \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}. \end{aligned} \quad (15)$$

The currents are

$$\begin{aligned} J_z^1 &= \frac{r^2}{1+r^2} (-2 \cos \phi \partial_z \theta + \sin \phi \sin 2\theta \partial_z \phi), \\ J_z^2 &= \frac{r^2}{1+r^2} (-2 \sin \phi \partial_z \theta - \cos \phi \sin 2\theta \partial_z \phi), \\ J_z^3 &= \frac{r^2}{1+r^2} (-2 \sin^2 \theta \partial_z \phi). \end{aligned} \quad (16)$$

Gauging of the model with respect to the subgroup $SO(3)$ requires three gauge fields A_μ^a . The right fermions, which are the superpartners of the fields x_i , are minimally coupled to the gauge fields and are described by the action

$$I_F^R = \frac{k}{4\pi} \int_\Sigma d^2 z \text{Tr} \psi_i D_{\bar{z}}^{ij} \psi_j, \quad (17)$$

with $D_{\bar{z}}^{ij} = \partial_{\bar{z}} \delta^{ij} + A_{\bar{z}}^a (T^a)^{ij}$. This action produces the well-known chiral nonabelian anomaly which in two dimension is equal to [7] (not to be confused with the abelian γ^{D+1} anomaly):

$$\delta I_F^R = \frac{1}{4\pi} \int_\Sigma d^2 z \text{Tr} u (\partial_z A_{\bar{z}} - \partial_{\bar{z}} A_z). \quad (18)$$

The left fermions are minimally coupled to gauge fields with arbitrary charges. Strictly speaking, bosonizing this fermionic theory is a very non-trivial problem, because, in principle, we must ensure conservation of all the symmetries of the fermionic theory; moreover, we must guarantee that all the correlation functions in both theories coincide. Nevertheless, for the purpose of this paper, we can use the arbitrariness in construction of the heterotic coset model. If we postulate that the fundamental characteristic of a model is anomaly, then, since the expression for anomaly of the fermionic part of the original model is independent of fermionic fields, we can introduce a bosonic theory yielding the same anomaly. In the nonabelian case, we would like to have something like the bosonic action (4), but this is no longer possible because the strength tensor now includes the commutator of the gauge fields. Moreover, it may be checked that there is no modification of the action (4) giving the desired form of the anomaly. A suitable possibility for bosonization of the fermionic sector is to take a gauged WZW action. The problem of this choice is that for nonabelian subgroups it is impossible to express the final background in a form in which backgrounds depend only on what we would like to call the target space geometry. Actually, we end up with a mixture of physical space coordinates and "internal" coordinates, because the bosonic fields, that we introduced to replace the fermions, appear in backgrounds as internal coordinates.

To obtain backgrounds, we must integrate out the gauge fields in the one-loop approximation, i.e. in the $k \rightarrow \infty$ limit (or in the $Q^2 \rightarrow \infty$ limit). In this limit, we can extract a part of the metric that depends only on "physical" coordinates which enter the original WZW model ⁴.

In the case under consideration, for bosonic action giving the same gauge anomaly as the fermionic sector, we take the gauged WZW model defined on the coset $SO(3)/SO(3)$. Our choice is dictated by the fact that this coset has three bosonic fields (we need three right fermions) and contains the gauge

⁴If we consider the action (5) as a 3D action on its own, independent of its origin, then, since the 3D backgrounds are independent of the third coordinate (the field Φ), we can make the standard dimensional reduction leading to 2D backgrounds which differ from those in (11). As noted in [4], this way of doing is not correct because the 3D action was obtained by bosonizing the fermions and, as a result, the constants k and Q are connected. Note, however, that backgrounds obtained by these two ways have similar structure and differ only by numerical coefficients.

subgroup ⁵. Here we see how the ambiguity in the choice of the bosonized fermionic theory can be used. The total action is:

$$\begin{aligned}
I^{total} &= I_{RGWZW} + I(\tilde{g}) + \frac{1}{2\pi} \int_{\Sigma} d^2z \left((a_z^i \tilde{J}_z^i - a_z^i Q^i \tilde{J}_z^i) \right. \\
&\quad \left. + a_z^i a_z^j (Q^j Tr T^j \tilde{g} T^i \tilde{g}^{-1} - \frac{1}{2} T^i T^j (1 + Q^i Q^j)) \right)
\end{aligned} \tag{19}$$

where there is no summation over the index of the charge Q_i . Here $\tilde{g} \in SO(3)$ is represented by the $SO(4)$ matrix of the form

$$\begin{pmatrix} 1 & 0 \\ 0 & \tilde{g} \end{pmatrix}$$

and the WZW action is defined as in (1). Now we are going to specialize to a particular case. The anomaly cancellation condition is

$$k Tr T^a T^b + Tr T^a T^b - Q^a Q^b Tr T^a T^b = 0 \tag{20}$$

A particular solution is given by $Q^1 = Q^2 = Q^3 = Q$. In this case, the relation between k and Q is $k = Q_+ Q_-$. Eliminating the gauge field in the semiclassical limit, we obtain

$$\begin{aligned}
I^{total} &= kI(g) + I(\tilde{g}) \\
&\quad - \frac{k}{2\pi} \int_{\Sigma} d^2z \left(\frac{Q}{k^2} \tilde{J}_z^a \Lambda_{ab} \tilde{J}_z^b + \frac{Q}{k} \tilde{J}_z^a \Lambda_{ab} J_z^a \right)
\end{aligned} \tag{21}$$

and

$$\Lambda^{ab} = Tr \left(\frac{1}{2k} T^a T^b (1 + k + Q^2) - \frac{Q}{k} T^b \tilde{g} T^a \tilde{g}^{-1} \right). \tag{22}$$

Here the matrix Λ_{ab} is the inverse of Λ^{ab} . Nonabelian structure of the action (21) is made explicit by writing the analog of the formula (9), which in this case is:

$$\begin{aligned}
I^{total} &= kI(g) + I(\tilde{g}) \\
&\quad + \frac{k}{2\pi} \int_{\Sigma} d^2z \left(\frac{Q Q_-}{k^2} \tilde{J}_z^a \Lambda_{ab} \tilde{J}_z^b \right.
\end{aligned}$$

⁵ Since this theory has anomalies, it cannot be interpreted as a topological theory; in fact, the whole theory is defined on the coset $SO(4) \otimes SO(3)/SO(3)$.

$$\begin{aligned}
& - \frac{Q^2}{k^2} (\tilde{J}_{\bar{z}}^a + \frac{k}{2Q} I_{\bar{z}}^a) \Lambda_{ab} (\tilde{J}_z^b + \frac{k}{2Q} J_z^b) \\
& - \frac{Q}{2k} (\tilde{J}_{\bar{z}}^a \Lambda_{ab} J_z^b - I_{\bar{z}}^a \Lambda_{ab} \tilde{J}_z^b) \\
& + \frac{1}{4} I_{\bar{z}}^a \Lambda_{ab} J_z^b,
\end{aligned} \tag{23}$$

where $I_{\bar{z}}^a = \text{Tr} T^a g^{-1} \partial_{\bar{z}} g$. Comparing with the calculations of the previous section, we see that now the role of $\partial_{\bar{z}} \Phi$ is played by $\tilde{J}_{\bar{z}}^a$. From equation (22) we have that in the large k approximation

$$\Lambda^{ab} = \text{Tr} T^a T^b + O\left(\frac{1}{\sqrt{k}}\right)$$

In this limit, the part depending on \tilde{g} decouples from the part depending on g , leaving the structure independent of the "inner" coordinates.⁶ As in section 2, we can associate to (23) the heterotic model with the action

$$\begin{aligned}
I^{total} &= \frac{k}{2\pi} \int_{\Sigma} d^2 z \left(G_{ij} \partial_{\mu} x^i \partial^{\mu} x^j \right. \\
&+ \text{Tr} \lambda_R (\partial_{\bar{z}} \lambda_R + [A_i \partial_{\bar{z}} x^i, \lambda_R]) \\
&+ \text{Tr} \lambda_L (\partial_z \lambda_L + Q [A_i \partial_z x^i, \lambda_L] \\
&\left. + F(g) \lambda_R \lambda_L \lambda_L \right).
\end{aligned} \tag{24}$$

Here we used the fact that the right fermions are the superpartners of the group coordinates x_i . This implies that the right gauge field may be introduced as $A_i^a : \text{Tr} T^a g^{-1} \partial_z g = A_i^a \partial_z x^i$. Explicit expressions for the gauge fields are:

$$\begin{aligned}
A_{\phi}^1 &= \frac{r^2}{1+r^2} (\sin \phi \sin 2\theta), \\
A_{\phi}^2 &= \frac{r^2}{1+r^2} (-\cos \phi \sin 2\theta), \\
A_{\phi}^3 &= \frac{r^2}{1+r^2} (-2 \sin^2 \theta), \\
A_{\theta}^1 &= \frac{r^2}{1+r^2} (-2 \cos \phi),
\end{aligned}$$

⁶ In particular, this is the reason why we did not need the explicit form of the currents \tilde{J}^a .

$$\begin{aligned}
A_\theta^2 &= \frac{r^2}{1+r^2}(-2\sin\phi), \\
A_\theta^3 &= 0, \\
A_r^i &= 0.
\end{aligned} \tag{25}$$

Here $i = 1, 2, 3$. As before, the antisymmetric tensor is zero. Using the same procedure as in the previous section, we obtain the following nonzero elements of the metric:

$$\begin{aligned}
G_{rr} &= \frac{1}{(1+r^2)^2} \\
G_{\theta\theta} &= \frac{r^2}{1+r^2} - \frac{r^4}{(1+r^2)^2} \\
G_{\phi\phi} &= \sin^2\theta \left(\frac{r^2}{1+r^2} - \frac{r^4}{(1+r^2)^2} \right).
\end{aligned} \tag{26}$$

Changing the coordinate $r \rightarrow \tan(r/2)$, we obtain

$$\begin{aligned}
G_{rr} &= 1 \\
G_{\theta\theta} &= \sin^2 r \\
G_{\phi\phi} &= \sin^2 r \sin^2 \theta.
\end{aligned} \tag{27}$$

This is the S^3 metric which emerges because of the relation between the algebras $so(4) \sim so(3) \oplus so(3)$. Models with this metric were discussed before [8]. We can consider this theory as the spacial part of a cosmological solution in a fashion similar to that of [8], where for the spacial part was taken the WZW model defined on the group $SO(3)$ and the time coordinate was a free field with the corresponding conformal field theory.

4 Noncompact Cosets

In this section we apply technique described in the preceding sections to the noncompact cosets $SO(2, 1)/SO(1, 1)$ and $SO(3, 2)/SO(3, 1)$. Here we omit some of the details discussed earlier. For the group $SO(2, 1)$, fixing the gauge as in sect.2, we obtain the following form of element

$$t = \begin{pmatrix} b & (b+1)\hat{X}^\nu \\ -(b+1)\hat{X}_\mu & \eta_\mu^\nu - (b+1)\hat{X}_\mu\hat{X}^\nu \end{pmatrix},$$

here \hat{X}^μ is a row $(x^1, -x^2)$, \hat{X}_μ is a column composed of the same elements, $b = (1 - x^2)/(1 + x^2)$. The indices are contracted with the metric $\eta_{\mu\nu} = \text{diag}(1, -1)$. As usual, for the noncompact cosets, we change the sign of the level k in the gauged WZW action. The anomaly cancellation condition remains the same, because the sign of $\text{Tr} T^2$ also changes. The generator T is

$$T = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

Changing the coordinates

$$x^1 = r \sinh t \quad x^2 = r \cosh t, \quad (28)$$

and repeating the steps of described in section 2, we obtain the following background

$$\begin{aligned} ds^2 &= dr^2 - \sinh^2 r dt^2 \\ A_t &= 2 \sinh^2 \frac{r}{2} \quad A_r = 0. \end{aligned} \quad (29)$$

This background can be interpreted as the electrically charged 2d black hole (for a more general construction see [4]).

The other coset of interest is $SO(3, 2)/SO(3, 1)$. Written in spherical coordinates

$$\begin{aligned} x^1 &= t \sinh r \\ x^2 &= t \cosh r \sin \theta \\ x^3 &= t \cosh r \cos \theta \sin \psi \\ x^4 &= t \cosh r \cos \theta \cos \psi, \end{aligned} \quad (30)$$

the part of the metric calculated from $I(g)$ is

$$ds_0^2 = -\frac{8}{(t^2 - 1)^2} dt^2 + \frac{8t^2}{t^2 - 1} (dr^2 + \sinh^2 r (d\theta^2 + \sin^2 \theta d\psi^2)). \quad (31)$$

The generators of the gauged subgroup are L_{ij} with $i, j = 2, 3, 4, 5$, where L_{ij} is the generator of the (psuedo)rotation in the plane (i, j) . The gauge fields

are calculated from the currents as in the previous sections; here we present the final expressions for the currents

$$\begin{aligned}
J_z^0 &= \frac{t^2}{t^2 - 1} (-2 \sin \theta \partial_z r + \sinh 2r \cos \theta \partial_z \theta) \\
J_z^1 &= \frac{t^2}{t^2 - 1} (-2 \cos \theta \sin \psi \partial_z r - \sinh 2r \sin \theta \sin \psi \partial_z \theta + \sinh 2r \cos \theta \cos \psi \partial_z \psi) \\
J_z^2 &= \frac{t^2}{t^2 - 1} (-2 \cos \theta \cos \psi \partial_z r - \sinh 2r \sin \theta \cos \psi \partial_z \theta - \sinh 2r \cos \theta \sin \psi \partial_z \psi) \\
J_z^3 &= \frac{t^2}{t^2 - 1} (2 \cosh^2 r \sin \psi \partial_z \theta + \cosh^2 r \sin 2\theta \cos \psi \partial_z \psi) \\
J_z^4 &= \frac{t^2}{t^2 - 1} (2 \cosh^2 r \cos \psi \partial_z \theta + \cosh^2 r \cos 2\theta \cos \psi \partial_z \psi) \\
J_z^5 &= \frac{t^2}{t^2 - 1} (2 \cosh^2 r \cos^2 \theta \partial_z \psi)
\end{aligned} \tag{32}$$

As in the previous cases (symmetric spaces), the final metric takes the form of the initial one after reparametrization of variables. The metric we obtained is that of the anti-de Sitter space. This shows a substantial difference from the treatment of the $SO(3, 2)/SO(3, 1)$ coset model in the diagonal vector gauging of the WZW model [8], where the low energy limit of conformal field theory does not display any symmetry and cannot be interpreted as a reasonable cosmological model.

5 Conclusions

We have constructed backgrounds which were obtained by applying heterotic coset model technique to cosets with nonabelian subgroups. As a further generalization of this construction, we can consider asymmetric gauging of both the left and right gauged subgroups.

Since in the present approach string backgrounds are deduced from conformal field theories, this ensures conformal symmetry of the corresponding string theory. More exactly, the required backgrounds are obtained by integrating out the gauge fields. If this procedure could be carried out exactly, one would obtain exact backgrounds valid in all orders in α' . Otherwise, integration over the gauge fields is performed in a certain approximation

yielding backgrounds verifying the β -equations in the corresponding order in α' . Taking the limit $k \rightarrow \infty$ one obtains backgrounds which are solutions of the leading-order β -equations; however, in the limit $k \rightarrow \infty$ corresponding to $\alpha' \rightarrow 0$, the effective action reduces to its leading part.

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